THE GRAVITATIONAL SUPPRESSION HYPOTHESIS:
DYNAMICAL ANALYSIS IN THE SMALL VELOCITY REGIME

Christian Marinoni[1] and Federico Piazza[2,3]

[1] Laboratoire d'Astrophysique de Marseille, Traverse du Siphon B.P.8, 13376 Marseille, France
[2] Dipartimento di Fisica, Università di Milano Bicocca, Piazza delle Scienze 3, I-20126 Milano, Italia
[3] Institute of Cosmology and Gravitation, University of Portsmouth, Portsmouth PO1 2EG, UK
Christian.Marinoni@oamp.fr, Federico.Piazza@mib.infn.it
Draft version February 2, 2008

ABSTRACT

In a previous paper we have proposed a non–Newtonian, phenomenological description of the effective gravitational force acting between exotic dark matter and baryons, which was shown to fit well the kinematics in the inner regions of low surface brightness, gas–rich galaxies. The Gravitational Suppression (GraS) scheme is, admittedly, an *ad-hoc* model; it does however successfully address interesting cosmological issues. Here, we test the potential generality of the GraS concept. GraS predictions on motions on scales measurable in dwarf spheroidal galaxies and in the solar neighborhood show significant departures from those of Newtonian mechanics. By analyzing such systems, we find that the paradigm of a universal GraS potential cannot be rejected. Indeed, compared to Newtonian predictions, GraS provides a better description of data, when realistic dark matter density profiles are considered.

Subject headings: Cosmology: dark matter — Galaxies: dwarf — Galaxies: halos — Galaxy: solar neighborhood — Gravitation — Stellar dynamics

1. INTRODUCTION

Dark matter (DM) has proved to be an essential component of the self–consistent cosmological framework, within which observations on a wide range of cosmic scales are generally interpreted. Despite its pervasive role, however, its nature is still elusive and its properties mostly speculative. In an attempt to gain insight of its fundamental characteristics, and motivated by the apparent conflicts faced by Cold Dark Matter (CDM) scenarios when compared with observations on small cosmological scales (e.g., Sellwood & Kosowsky 2001; Ostriker & Steinhardt 2003), we have recently hypothesized that the gravitational attraction between DM and baryons may be suppressed on the kpc scale (Piazza & Marinoni 2003, hereafter paper I). The gravitational suppression (GraS) hypothesis was suggested by the simple observation that all the various inconsistencies between theory and observations appear to be characterized by a common physical length. GraS minimally modifies the existing paradigm, i.e. it does not alter large–scale interactions or baryon and DM self–dynamics.

In paper I we modeled the effective force acting between visible and dark particles by adding a short–range Yukawa contribution to the standard Newtonian potential. We also investigated the consistency and universality of such an empirical gravitational paradigm, fixing the model parameters using observations of the rotation curves of low surface brightness (LSB), gas–rich galaxies, the cores of which are thought to be DM–dominated. The scale length λ over which the Yukawa contribution is effective is about one kpc, and its strength on small distances is equal and opposite to the Newtonian one. In other words, the inner dynamics of LSB galaxies seems to point towards a "total suppression" scenario, whereby DM is completely oblivious of normal matter in its immediate vicinity.

The analysis of paper I was mainly concerned with rotationally supported disk galaxies having circular velocity of order ~ 100 km/s. Here, we investigate consequences and predictions of the proposed gravity model on a class of systems having different matter distributions and dynamical symmetries, with characteristic velocities nearly a factor 10 smaller than those of typical LSB galaxies. In particular we aim:

a) to assess the ability of the model in describing the kinematics of dwarf spheroidal (dSph) galaxies, i.e. in a physical regime of spherically symmetric, pressure-supported, low-velocity, virialized systems. This study is motivated by recent claims (e.g., Lokas 2002) according to which a universal Navarro, Frenk & White (1997, NFW) DM profile can reproduce the shape of the velocity dispersion profiles of dSph galaxies only if a substantial (and poorly justified) amount of tangential anisotropy in the velocity distribution of dSph stars is assumed. By confronting kinematical data of dSph galaxies with the GraS predictions, we wish to verify whether the latter achieves the same interpretative success as with LSB galaxies

b) to check the ability of GraS in describing both kinematics and space distribution of stars perpendicular to the galactic disk in the vicinity of the Sun (e.g., Oort 1932; Siebert, Bienaymé, & Soubiran 2003). Since the presence of a spheroidal, non baryonic, DM halo embedding the thin galactic disk affects the density distribution of stars near the galactic plane (Bahcall 1984), it is imperative to quantify the dynamical effects on stellar motions which are the consequence of GraS. It has been noted (Kuijken & Gilmore 1989) that an intrinsically different non-Newtonian theory (Modified Newtonian Dynamic: MOND, by Milgrom 1983), while capable of explaining the flat rotation curves of galaxies without requiring the presence of a dark matter halo, leads to inconsistent predictions on the vertical component of the gravitational acceleration experienced by solar neighborhood disk stars. Does GraS score any better?

1

2. THE MODEL

As in paper I, we model the proposed short-range modification of gravity by adding to the standard Newtonian potential a repulsive Yukawa contribution which is active only in the mixed (dark–visible) sector. We can then write the total gravitational potential acting on baryons as $\Phi = \Phi_N - \Phi_Y$, where Φ_N satisfies the usual Poisson equation

$$\nabla^2 \Phi_N = 4\pi G \left(\rho_B + \rho_D\right) \tag{1}$$

where ρ_B and ρ_D are respectively the baryonic and DM densities. The Yukawa contribution Φ_Y typically dies out exponentially with a radial scale length λ, according to

$$(\nabla^2 - \lambda^{-2}) \Phi_Y = 4\pi G \rho_D. \tag{2}$$

Notice that Φ_Y is sourced by the DM energy density only. The resulting gravitational potential between a DM and a baryonic particle at a distance r from each other goes like $(e^{-r/\lambda} - 1)/r$. In what follows we assume $\lambda = 1$ kpc as suggested by the study of LSB galaxies in paper I.

3. MASS DISTRIBUTION AND DYNAMICS IN DSPH GALAXIES

Dwarf spheroidal galaxies, being small and DM dominated, provide a test for the validity of GraS in a low-velocity, shallow potential regime. Since the typical luminous dimension of a dSph is ~ 1 kpc, its baryonic dynamics is expected to be strongly influenced by the specific form of our gravitational potential, which is significantly dampened with respect to the pure Newtonian one on these scales. Thus: given a realistic DM density profile, is the baryon dynamics derived within the GraS paradigm in agreement with the observations of the stellar velocity dispersion in dSph galaxies?

The stellar radial velocity dispersion profile σ_r induced by the total potential Φ can be obtained by solving the Jeans equation for a pressure supported spherical system

$$\frac{d}{dr}(\rho_B \sigma_r^2) + 2\frac{\beta}{r}\rho_B \sigma_r^2 + \rho_B \frac{d\Phi}{dr} = 0 \tag{3}$$

where $\beta = 1 - \sigma_\theta^2(r)/\sigma_r^2(r)$ accounts for the anisotropy in the stellar velocity field.

The luminous component of a dSph galaxy is well described by the Sérsic profile (Sérsic 1968; Ciotti 1991)

$$I(r) = I_0 \exp[-(r/r_0)^{1/m}] \tag{4}$$

with exponential index ~ 1 (but see Caldwell 1999) where $I(r)$ is the surface brightness. The baryonic energy density $\rho_B(r)$ can be obtained from $I(r)$ by deprojection:

$$\rho_B(r) = -\frac{\Upsilon_*}{\pi} \int_r^\infty \frac{dI(t)}{dt} \frac{dt}{\sqrt{t^2 - r^2}} \tag{5}$$

(see Mazure & Capelato (2002) for an explicit analytical solution of this integral) where Υ_* is the baryonic mass-to-light ratio. To first approximation, Υ_* is independent on the radius for the objects considered. The solution (5) can be applied to equation (3) and in the inversion of equation (1) to obtain Φ_N. The DM component ρ_D is also needed, as it contributes to both the Newtonian (1) and the Yukawa (2) potentials. We describe ρ_D via a generalized NFW profile of inner slope index γ

$$\rho_D(r) = \frac{\rho_0\,\delta}{(r/r_s)^\gamma(1 + r/r_s)^{3-\gamma}}, \tag{6}$$

where ρ_0 is the critical density of the Universe, δ is a characteristic overdensity and r_s a radial scale parameter. According to N-body simulations, $\gamma = 1$ (Navarro, Frenk & White 1997) and $\gamma = 1.5$ (Moore et al. 1998) bracket the plausible range of variation of this parameter (Navarro et al. 2003).

The virial radius r_v of an object is that within which the mean density is $\Delta \simeq 200\rho_\circ \simeq 0.023(M/h^{-1}M_\odot)^{1/3}$ kpc. The radius r_s marks the transition from the outer r^{-3} to the inner $r^{-\gamma}$ density behavior. A concentration parameter for the DM halo $c = r_v/r_s$ can be defined, which in N-body simulations exhibits a mild dependence on the total mass of the halo. We have not found explicit results for objects of masses $< 10^{11} M_\odot$, so we extrapolate the results of Eke, Navarro & Steinmetz (2001) to small masses and use, for the NFW $\gamma = 1$ case, the formula

$$c_{\gamma=1} = 81 \left(\frac{M}{h^{-1}M_\odot}\right)^{-0.07}. \tag{7}$$

while, for the steeper profile $\gamma = 1.5$ we set $c_{\gamma=1.5} = (1/2)c_{\gamma=1}$ (Jing & Suto 2000).

Equation (2) in the case of spherical symmetry reduces to

$$\frac{d^2\Phi_Y}{dr^2} + \frac{2}{r}\frac{d\Phi_Y}{dr} - \frac{\Phi_Y}{\lambda^2} = 4\pi G\rho_D(r). \tag{8}$$

and in paper I an integral expression for Φ_Y in terms of ρ_D was outlined. However, for generic $\gamma \neq 1$ only numerical solutions of (2) can be found, while the solution of the Poisson equation (1) can always be given in closed analytic form for any value of γ. Once the total potential Φ is obtained, equation (3) can be solved for the radial velocity dispersion. A more useful quantity is the line-of-sight velocity dispersion σ_{los}, which is related to σ_r through (Binney & Mamon 1982)

$$\sigma_{los}^2(r) = \frac{2}{\Upsilon_* I(r)} \int_r^\infty \left(1 - \beta\frac{r^2}{t^2}\right) \frac{\rho_B(t)\sigma_r^2(t)t}{\sqrt{t^2 - r^2}} dt \tag{9}$$

and provides a direct comparison with observations.

4. MODELS VS. OBSERVATIONS

Spatially resolved kinematic data became recently available for two dSph galaxies: Fornax (Mateo 1997) and Draco (Kleyna 2002). Under the assumption that these galaxies are in dynamical equilibrium, as found by Mateo (1998); Piatek et al. (2002); Odenkirchen et al. (2001); Kleyna et al. (2002)), we can compare the observed velocity dispersions with those predicted by GraS. Assuming that the measurement errors of the dSph velocity profiles are normally distributed (possible systematics such as the influence of binary populations are negligible as shown for example by De Rijcke & Dejonghe (2002)), we compare the observations of σ_{los} with Equation (9) and constrain the scale parameters of the DM density profile via least square minimization. We also fix the parameters characterizing the stellar distribution (i.e. the baryons properties) as given by the observations, as shown in Table 1.
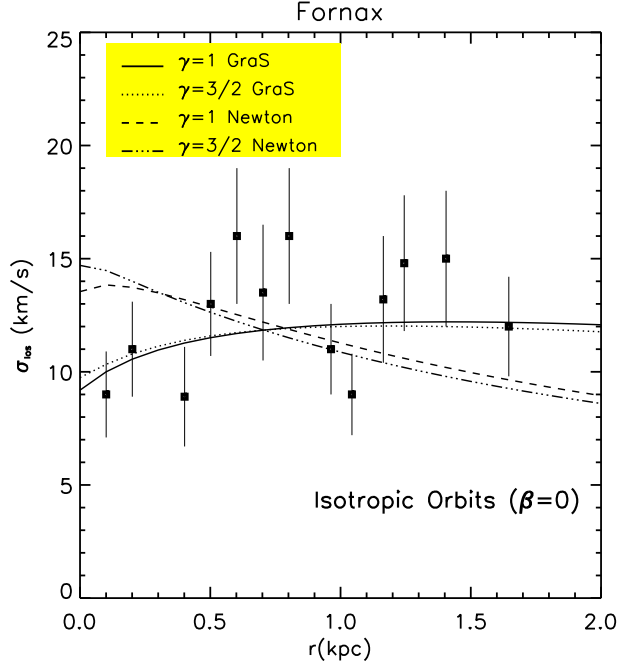
FIG. 1.— The observed velocity dispersion profile of Fornax (Mateo 1997) is shown together with four different best fitting velocity dispersion profiles inferred assuming isotropic orbits (β is kept fixed to 0 in the fit). Theoretical predictions are computed in the case of a DM inner density profile slope of 1 (Navarro, Frenk & White 1997) or 3/2 (Moore et al. 1998) and according to both the pure Newtonian and the GraS paradigms, as labeled. Error bars indicate 1σ uncertainties.

FIG. 2.— The same as in fig. 1 but for the Draco velocity dispersion profile. Data are from Kleyna et al. (2002)

Numerical minimization is performed assuming as free parameters of the model the virial mass M of the dark halo embedding the dSph and the velocity anisotropy parameter β which, for simplicity, is assumed to be independent of the radial scale. Thus, our results do not rely on any *a-priori* assumed anisotropic model: the velocity anisotropy is self-consistently evaluated by our fitting scheme. The resulting best fitting values are presented in Table 2.

Our results may be described as follow:

a) the predicted GraS gravitational acceleration generated by a two component fluid of baryons (distributed according to the Sersic profile) and DM (distributed according to the generalized NFW profile) is in agreement with the stellar velocity dispersions observed in the dSph galaxies Fornax and Draco, if the stellar orbits are isotropic as shown in Figs. 1 and 2

b) the best fitting velocity anisotropy parameter $\beta \sim 0$ derived within the GraS gravitational paradigm is consistent with the results of N-body simulations of DM halos (e.g., Thomas et al. 1998). Table 2 shows that a Newtonian analysis of the kinematics within a cuspy, NFW DM profile agrees with the observations only if the existence of a strong tangential anisotropy in the dSph stellar orbits is assumed (see also Lokas (2002)). The amount of tangential anisotropy needed to fit the profiles is, however, unphysically large if confronted with observations of stars in the solar neighborhood (e.g., Chiba & Berrs 2000) or the evidence that elliptical galaxies are moderately, radially anisotropic (e.g., Gerhard et al. 2001)
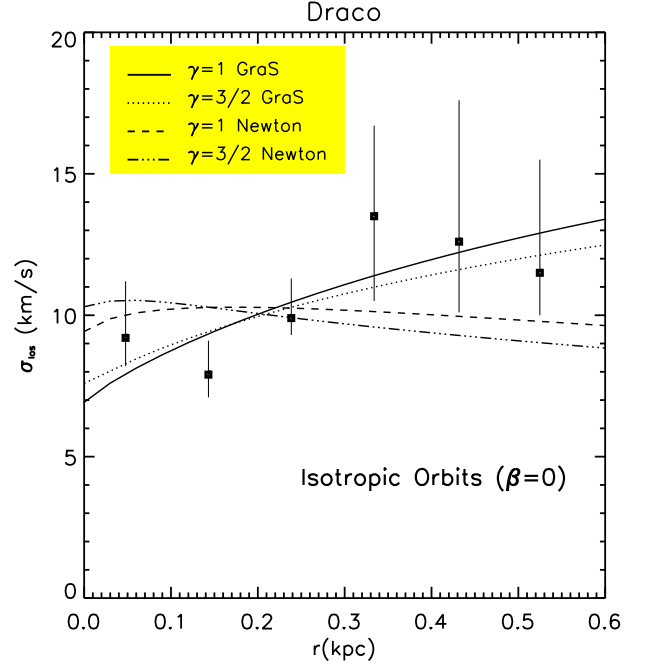
c) the best fitting mass within the virial radius depends on the assumed concentration parameter (eq. 7). For realistic values of c, and for all gravity models, the mass of Fornax and Draco is larger than commonly assumed (see Stoehr et al. (2002) for a similar conclusion). Since in pressure supported systems there is a well known degeneracy of density profiles versus velocity anisotropy (Binney & Tremaine 1987), the best fitting masses present a non negligible degree of covariance with respect to the best β estimate. The more negative the β parameter (as in the Newtonian case), the bigger is the inferred mass (see Table 2). Moreover, the steeper the cusp of the profile (higher γ), the more tangential is the velocity distribution required to fit the data, in agreement with the finding of Lokas (2002). We also note that our results for M_t/L_V (the mass-to-light ratio inside the dSph tidal radius r_t) agree in order of magnitude with other independent estimates based on Newtonian analysis (e.g., Kleyna et al. (2001,2002); Odenkirchen et al. (2001))

d) dSph galaxies have low densities and therefore they lie in the regime of small accelerations for which MOND provides an alternative interpretation of the observed kinematics, without reliance on the DM hypothesis. However, Lokas (2001) and Kleyna et al. (2001) found that the best fitting values of $a_\circ$, the characteristic acceleration scale ($a_0 \sim 10^{-8}$ cm s^{-2}) below which the laws of Newtonian dynamics change, are different for different dSph (low for Fornax, much higher than expected in the case of Draco). They concluded that the Draco poses a serious problem for MOND. At variance, the GraS parameters are stable and robust even in the case of these two systems which have similar velocity dispersion but very different luminosities.

5. THE SOLAR NEIGHBORHOOD DYNAMICS

We next test GraS predictions in the sub-kiloparsec, small velocity regime of stellar kinematics in the direction z perpendicular to the galactic disk. In the vicinity of the disk the gravitational potential can be locally described by (Bahcall 1984)

$$\frac{\partial^2 \Phi_N}{\partial z^2} = 4\pi G(\rho_B^{\rm disk} + \rho_D^{\rm halo}). \qquad (10)$$

In what follows, derivations with respect to z and r are meant while keeping the other coordinates – cylindrical and spherical respectively – fixed. Moreover, equation (10) is derived under the assumption that the rotation curve is flat ($\rho_D^{\rm halo} \propto r^{-2}$) which is, to a good approximation, a valid assumption at our position in the Galactic disk. We also note that in the solar neighborhood the halo density is about one tenth of the disk baryonic density (Gates, Gyuk & Turner 1995). As a consequence, the halo effects on the nearby stellar dynamics can be treated as a perturbation, approximated by expanding in $\epsilon \equiv \rho_D^{\rm halo}/\rho_B^{\rm disk}$ (Bahcall 1984). The dynamics of a single isothermal population of density ρ with velocity dispersion σ_z is then described by the first moment of the Boltzmann equation $\sigma_z^2 \partial\rho/\partial z = -\rho\,\partial\Phi/\partial z$.

We next estimate the correction terms introduced by GraS into eq. (10). Note that, since Φ_Y is spherically symmetric,

$$\frac{\partial^2 \Phi_Y}{\partial z^2} = \frac{1}{r}\frac{d\Phi_Y}{dr} + \mathcal{O}\left(\frac{z}{r}\right)^2 \frac{d^2\Phi_Y}{dr^2}, \qquad (11)$$

where the height from the disk z is small with respect to our distance r from the center of the Galaxy. In order to estimate the first term on the right hand side of (11)

we use equation (8). Since the distance of the Sun from the center of the Galaxy is about 8.5 kpc, we expand the solution in the $r \gg \lambda$ limit to obtain

$$\Phi_Y(r) = -4\pi G\lambda^2 \rho_D^{\rm halo}(r)\left[1 + \mathcal{O}\left(\frac{\lambda}{r}\right)^2\right]. \qquad (12)$$

Deviations from the Newtonian equation (10) are therefore expected at order $(\lambda/r)^2$ in the halo density contribution. For instance, in the isothermal approximation $\rho_D^{\rm halo} \propto r^{-2}$, we obtain

$$\frac{\partial^2 \Phi}{\partial z^2} \simeq 4\pi G\rho_B^{\rm disk}\left[1 + \epsilon\left(1 - \frac{2\lambda^2}{r^2}\right)\right]. \qquad (13)$$

At the Sun's location, the correction to the Bahcall parameter ϵ is thus negligible and of order $\sim 3\%$. We conclude that GraS is fully compatible and does not compromise our current understanding of the kinematics in the solar neighborhood.

GraS offers a phenomenologically viable paradigm with which to reconcile mismatches between observations and CDM predictions on small cosmological scales.

REFERENCES

Bahcall, J. N. 1984, ApJ, 276, 156
Binney, J., & Mamon, G. A. 1982, MNRAS, 200, 361
Binney, J., & Tremaine, S. 1987, Galactic Dynamics, Princeton University Press
Chiba, M., & Beers, T. C. 2000, AJ, 119, 2843
Ciotti L., 1991, A&A, 249, 99
De Rijcke S., and Dejonghe H. 2002, MNRAS, 329, 829
Eke V. R., Navarro J. F., & Steinmetz M. 2001, ApJ, 554, 114
Gates E. I., Gyuk G. & Turner M. S. 1995, ApJ, 449, L123
Gerhard, O., Kronawitter, A., Saglia, R. P., & Bender, R. 2001, AJ, 121, 1936
Kleyna J. T., Wilkinson M. I., Evans N. W. & Gilmore G. 2001, ApJ, 563, L115
Kleyna J. T., Wilkinson M. I., Evans N. W., & Gilmore G. 2002, MNRAS, 330, 792
Kuijken, K., & Gilmore G. 1989, MNRAS, 239, 605
Jing Y. P. & Suto Y. 2000, ApJ, 259, L69
Lokas, E. L. 2002, MNRAS, 333, 697
Mateo, M., Olszewski, E. W., Welch, D. L., Fisher, P., Kunkel, W. 1991, AJ, 102, 914
Mateo, M. 1997 in ASP Conf. Ser. Vol. 116, The Nature of the Elliptical Galaxies, ed. M. Arnaboldi et al. (San Francisco: ASP), 259
Mateo, M. 1998, ARA&A 36, 435

Mazure, A., & Capelato, H. V. 2002, A&A, 383, 384
Milgrom, M., 1983, ApJ, 270, 365
Moore B., Governato, F., Quinn, T., Stadel, J., Lake G. 1998 ApJ, 499, L5
Navarro J. F., Frenk C. S. & White S. D. 1997 ApJ, 490, 493
Navarro J. F. et al. 2003, MNRASsubmitted, astro-ph/0311231
Odenkirchen, M., et al. 2001, AJ, 122, 2538
Oort, J. H. 1932, Bull. Astr. Inst. Netherlands, 6, 249
Ostriker, J. P., & Steinhardt, P. 2003, Science 300, 1909
Piatek, S., Pryor, C., Armandroff, T. E., & Olszewski, E. W. 2002, AJ, 123, 2511
Piazza, F. & Marinoni, C. 2003, Phys. Rev. Lett., 91, 141301
Pryor, C. 1996, in ASP Conf. Ser., 92, Formation of the Galactic Halo...Inside and Out, ed. H. Morrison & A. Sarajedini (San Francisco: ASP), 424.
Sellwood, J. A., & Kosowsky, A. 2001, ASP Conf. Proc., 240. Ed. J. E. Hibbard, M. Rupen, & J. H. van Gorkom (San Francisco: ASP), 311
Sérsic, J. L. 1968, Atlas de Galaxies Australes, (Cordoba: Observatorio Astronomico)
Siebert, A., Bienaymé, O., & Soubiran, C. 2003, A&A, 399, 531
Stoehr, F., White, S. D. M., Tormen, G., & Springel, V. 2002, MNRAS, 335, 84
Thomas, P. A. et al. 1998, MNRAS, 296, 1061

TABLE 1
DWARF PARAMETERS

Parameters	Fornax	Draco
r_0(kpc)	0.35	0.18
r_t(kpc)	2.85	0.95
m	1	1.2
distance (kpc)	138	82
$L_V(L_\odot)$	$1.55 \cdot 10^7$	$2.6 \cdot 10^5$
$\Upsilon_*(M_\odot/L_\odot)$	2.5	2.5

Note. — All data are taken from Mateo (1998) except the Sersic scale radius, the exponential index m of the luminosity density profile and the tidal radius of Draco which have been recently re-estimated by Odenkirchen et al. (2001) using SDSS data. The baryonic mass-to-light radius represents the average value for a typical globular-cluster star population (e.g., Mateo et al. 1991; Pryor 1996)

TABLE 2
BEST FITTING PARAMETERS

	DM inner slope	Gravity Model	β	M $(10^9 M_\odot)$	M_t/L_V $10^2 M_\odot/L_\odot$	χ^2_ν
Fornax	$\gamma = 1$	GraS	$0^{+0.2}_{-0.4}$	$7.5^{+3}_{-2.5}$	0.65	0.96
	$\gamma = 3/2$	GraS	$-0.1^{+0.2}_{-0.2}$	9.5^{+4}_{-3}	0.65	0.95
	$\gamma = 1$	Newton	$-1.5^{+0.9}_{-1.8}$	$1.1^{+0.4}_{-0.4}$	0.23	0.91
	$\gamma = 3/2$	Newton	$-3^{+0.7}_{-0.8}$	13.6^{+5}_{-4}	0.76	0.92
Draco	$\gamma = 1$	GraS	$0.2^{+0.18}_{-0.23}$	30^{+25}_{-15}	16	0.78
	$\gamma = 3/2$	GraS	$0.2^{+0.11}_{-0.16}$	18^{+7}_{-6}	15	0.80
	$\gamma = 1$	Newton	$-1.4^{+0.9}_{-2.5}$	$1.7^{+1.1}_{-0.7}$	5	0.78
	$\gamma = 3/2$	Newton	$-6.4^{+1.5}_{-1.5}$	92^{+80}_{-50}	33	1.07

Note. — All the uncertainties represent 1σ errors